SCORE

State Collaborative on Reforming Education

A Review of Tennessee's Draft Mathematics Academic Standards

November 2015

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Introduction

The purpose of this review is to examine the October 2015 draft of the Tennessee Math Standards (Draft TMS) to determine whether they are high-quality standards that prepare students, over the course of their K–12 education careers, for success in credit-bearing college courses and quality, high-growth jobs. To complete this review, the State Collaborative on Reforming Education (SCORE) partnered with Achieve, an independent, nonpartisan, nonprofit education reform organization dedicated to working with states to raise academic standards and graduation requirements, improve assessments, and strengthen accountability. The organization, created in 1996 by a bipartisan group of governors and business leaders, is one of the nation's leading sources of expertise on K-12 academic standards.

When evaluating standards, Achieve uses a set of six criteria: rigor, coherence, focus, specificity, clarity/accessibility, and measurability. For the purposes of this analysis, the TMS were compared with the current Tennessee State Standards (TSS) for Mathematics and analyzed with respect to these criteria.

The current high school TSS specify the mathematics content all students should have mastered to become college and career ready. In addition to the standards for all students, the TSS also include additional standards that some students should know to prepare for advanced mathematics courses such as Calculus. These additional standards are identified in the TSS with a (+). Throughout this report, as in the TSS, we will refer to these additional standards as (+) standards. All TSS without a (+) symbol are intended to be common to all college- and career-ready students.

For grades 9–11, the TMS offer standards designated for two different course sequences: a Traditional sequence (Algebra I, Algebra II, and Geometry) and an Integrated sequence (Math I, Math II, and Math III). With the exception of a few variations in the two sequences, they generally have the same level of focus, rigor, and coherence. Tennessee is among the few states that require four years of math, including one year beyond Algebra II. To accommodate that requirement, the TMS include five different options for fourth-year courses: Bridge Math, Precalculus, Statistics, Applied Mathematical Concepts, and Calculus. This approach allows for at least eight different pathways for a four-year math student, with varying levels of rigor, coherence, and focus. The high school expectations intended for all students in the TMS provide a college- and career-ready set of standards, except for a few instances where they are somewhat less focused, rigorous, and/or coherent than Tennessee's current standards. Details about those areas can be found in this report and in the accompanying chart.

This report does not include close analysis of the fourth-year course standards and assumes that all Tennessee students would, minimally, be exposed to either the Traditional or the Integrated sequence of three courses in addition to one of the fourth-year courses. A complete review of the strengths and weaknesses of those course standards is advised in the future to ensure that students are prepared for various postsecondary pathways, including some that would require higher-level mathematics.

Review of Tennessee's Draft Mathematics Standards

This report provides a review of the draft of the TMS released in October 2015. The draft document provides grade-level standards for each of the grades from kindergarten through grade 8. In high school, course standards for the first three years are presented for both Traditional and Integrated sequences. The Traditional sequence consists of Algebra I, Algebra II, and Geometry, while the Integrated courses are simply titled Integrated Math I, Integrated Math II, and Integrated Math III. The draft also includes proposed standards for advanced fourth-year courses in Precalculus, Statistics, and Calculus, as well as for courses titled Bridge Math and Applied Mathematical Concepts. The TMS are structured around domains, clusters, and content standards, with the high school standards also being grouped by broader conceptual categories. The TMS are aligned to progressions, as indicated below. **The TMS progressions, listed on page 3 of the draft, reflect highly rigorous academic standards.**

TMS	Grade
Counting and Cardinality	K
Number and Operations in Base Ten	K–5
Number and Operations — Fractions	3–5
Ratios and Proportional Relationships	6–7
The Number System	6–8
Number and Quantity	9–12
Operations and Algebraic Thinking	K–5
Expressions and Equations	6–8
Functions	8
Algebra and Functions	9–12
Geometry	K–12
Measurement and Data	K–5
Statistics and Probability	9–12

The TMS include the same eight Standards for Mathematical Practice as those found in the TSS. These standards are included to recognize that success "requires that development of approaches, practices, and habits of mind be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop within their students" (page 8). The Standards for Mathematical Practice, as they first appear in the TMS are shown below:

TMS Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

On page 8 of the TMS, the eight Standards for Mathematical Practice have the same titles and numbering as in Tennessee's current standards. In the more detailed descriptors that appear on pages 9–12 of the TMS, however, the same practices are presented in a different order. Specifically, MP4 and MP6 have changed places and numbers, as have MP5 and MP7. However Tennessee chooses to number and order the practices, Achieve recommends that the same numbering and order be used consistently throughout the TMS document.

According to the TMS, "Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others and analyze and reflect on their own thought processes." To this end, the TMS include Literacy Skills for Mathematical Proficiency. These skills, with no direct counterpart in the TSS, include reading, vocabulary, speaking and listening, and writing and are summarized as follows:

TMS Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Given the mismatch in the numbering of the Mathematical Practices, the alignments of the Mathematical Practices to the Literacy Skills should be reviewed.

Since many of the high school standards in the TMS are used multiple times in the various courses, we have added to the TMS coding schema for this report to identify and compare the standards based on which course and sequence it is addressing. In most cases the standard is used multiple times but in the exact same form. For example, A.SSE.A.2 is addressed in four courses for grades 9–11: Algebra I and II and Integrated Math I and II. In each course the wording of the standard is exactly the same. We have indicated the course in the codes for the standard in the following way: **AI.**A.SSE.A.2, **AII.**A.SSE.A.2, **MI.**A.SSE.A.2, **MII.**A.SSE.A.2.

However, there are a few standards that are slightly changed when used in different courses. For example, A.CED.A.1 is addressed in five of the six TMS courses for grades 9–11 in the following ways:

TMS Traditional Sequence	TMS Integrated Sequence
AI.A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and rational and exponential functions.	**MI.**A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and rational and exponential functions.
	MII.A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and rational and exponential functions.
AII.A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems.	**MIII.**A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems.

By adding the course designation to the coding scheme for each standard, we are able to identify, for the purposes of this report, the course to which a standard is connected.

Rigor

Rigor refers to the intellectual demand of the standards. It is the measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in credit-bearing college courses without remediation and in entry-level, quality, high-growth jobs. Rigorous standards should reflect, with appropriate balance, conceptual understanding, procedural skill and fluency, and applications. For this report, Achieve compared the rigor of current Tennessee State Standards to that of the draft TMS.

In most respects, the TMS reflect comparable levels of rigor to the baseline standards for college- and career-readiness. As such, the emphasis on the three components of rigor, conceptual understanding, procedural skill and fluency, and application in the TMS has very nearly the same balance as that of the TSS. We do see a few instances, however, where the TMS have deemphasized an understanding, explanation, or proof associated with a given TSS standard. In the examples below, words or phrases in the aligned TSS were deleted with the result being the reduction of emphasis on conceptual understanding:

TSS Standard	**Draft TMS Standard**
1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	1.NBT.C.4. Add a two-digit number to a one-digit number and a two-digit number to a multiple of ten (within 100) using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	1.NBT.C.5. Mentally find 10 more or 10 less than a given two-digit number without having to count by ones.

2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	2.NBT.B.7. Add and subtract within 1,000 using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

In some cases, wording changes from prior standards could lead to reduced rigor. In these examples, for example, the word "prove" was changed to "use" or "recognize:"

TSS Standard	Draft TMS Standard
A.APR.4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2 can be used to generate Pythagorean triples.	**AII(MIII).**A.APR.C.4. Use polynomial identities to describe numerical relationships.
F.LE.1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	**AI(MI).**F.LE.A.1a. Recognize that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F.TF.8. Prove the Pythagorean identity sin^2(A) + cos^2(A) = 1 and use it to find sin(A), cos(A), or tan(A) given sin(A), cos(A), or tan(A) and the quadrant of the angle.	**AII(MIII).**F.TF.C.8. Use trigonometric identities to find values of trig functions.
G.C.1. Prove that all circles are similar.	**G.**G.C.A.1. Recognize that all circles are similar.

Below is another example from grade 5 in which the opening explanatory remarks in the TSS are replaced with the procedural "graph and label…" TMS 5.G.1 is limited to the first quadrant, which means that the graph uses only positive values. Given this limitation, the description of two perpendicular lines does not work.

TSS Standard	Draft TMS Standard
5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis,	5.G.A.1 Graph and label points using the first quadrant of the coordinate plane. Understand that the first number indicates the horizontal distance traveled along the *x*-axis from the origin and the second number indicates the vertical distance traveled along the *y*-axis with the convention that the names of the two axes and the coordinates correspond *(e.g., x-axis and x-coordinate, y-axis and y-coordinate).*

with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	

The TMS have lost the foundational understanding for this expectation. In addition, the change in wording caused the expressions "first number" and "second number" to have lost their connection to the coordinates in an ordered pair. This TMS change also inserts a limitation that requires only graphing points in the first quadrant, while the TSS do not.

Even though these sorts of changes are exceptions to the rule rather than the rule, consideration should be given to these types of changes that affect the rigor of the TMS.

Coherence

Coherence refers to how well a set of standards conveys a unified vision of the discipline, establishing connections among the major areas of study and showing a meaningful progression of content across the grades, grade spans, and courses.

The coherence of the draft TMS is comparable to that found in the TSS. There are a few subtle differences between the two sets, some of which arise from the challenge of creating two high school course sequences. It is helpful that the TMS include the Scope and Clarifications for each standard in each course.

One issue to consider is the TMS grade 8 handling of both similarity and congruence but also of dilations in connection to similarity. Dilations are addressed in grade 8 in both the TSS and draft TMS, but the connection to similarity is not a part of the TMS. It can be seen in the table below, that the foundation for understanding and using transformations is laid in the TMS with the inclusion of their 8.G.A.1 and 8.G.A.2. (Note: TMS 8.G.A.2 is coded as 8.G.3 in the TSS.) However, connecting that understanding of transformations to similarity and congruence has been omitted in the TMS decision not to include alignments to 8.G.2 and 8.G.4 in the TSS.

TSS Grade 8	Draft TMS Grade 8
8.G.1. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	8.G.A.1. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	There is no match in the TMS.

8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	8.G.A.2. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	There is no match in the TMS.

Tennessee students will likely miss the connection between dilations and a definition and understanding of similarity. The lack of coherence in these changes is amplified in that the cluster heading for TMS 8.G.A.2 claims to include understanding congruence and similarity:

> *8.G.A. Understand congruence and similarity using physical models, transparencies, or geometry software.*

Under this heading we would expect, at the very least, to see standards that address both similarity and congruence and, at best, a connection to the transformations that would be related to the "physical models, transparencies, or geometry software" mentioned in the heading.

The draft TMS address the Laws of Sines and Cosines in the high school course sequences standards (in the first three years). The TSS address this as a (+) standard (G.SRT.11), intended for those students who have an interest in math-related studies or careers. However, in the Geometry and Math II course standards, "understand and apply" has been changed to "recognize and use," with respect to the laws. In doing so, the TMS have removed the notion of understanding the laws, instead expecting only for students to recognize and apply them. Further, it is not clear what the draft standard means by "recognize" the Laws of Sines and Cosines. The intention of the TMS may be that students know *when* to use the Laws of Sines and Cosines. If that is the case, this wording should be made clearer. This standard appears again in the draft Precalculus TMS. In the Precalculus class, the standard uses "understand and apply." The coherence issue here is that understanding the laws comes after using them and only outside of the sequenced courses. It should also be noted here that G.SRT.10 (+), requiring proof of the laws, is not addressed in the TMS at all. This completely eliminates the requirement that students know that the Laws of Sines and Cosines are true and defendable.

TSS Geometry	Draft TMS Geometry	Draft TMS Precalculus
G.SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	**G(MII).**G.SRT.C.8b. Recognize and use the Law of Sines and the Law of Cosines to solve triangles in applied problems.	G.AT.A.6. Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Standards reviewers should also address inconsistent limits between the two course sequences. For example, consider A.CED.A.4: *Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.* The standard is addressed in Algebra I and in each of the three Integrated courses. The table below shows the scope of this standard at each level:

Algebra I	Integrated I	Integrated II	Integrated III
There are no assessment limits for this standard. The entire standard is assessed in this course.	i) Tasks are limited to linear equations. ii) Tasks have a real-world context.	i) Tasks are limited to quadratic, square root, cube root, and piecewise functions. ii) Tasks have a real-world context.	i) Tasks have a real-world context. ii) Tasks are limited to polynomial, rational, absolute value, exponential, or logarithmic functions.

Given that the Algebra I course is restricted to linear, quadratic, and exponential functions, there is a higher expectation for students in the Integrated sequence for this particular standard.

Similarly, there is a mismatch in expectation with respect to:

> A.REI.A.1 Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

This standard is addressed in Algebra I, Algebra II, and Integrated II and III. The students in the Traditional sequence, missing alignment to piecewise, square roots, and cube roots, will have a lower overall expectation to apply this standard. The table below shows the limitations for A.REI.A.1 at each level:

Algebra I	Algebra II	Integrated II	Integrated III
Tasks are limited to linear and quadratic equations.	Tasks are limited to simple rational or radical equations.	Tasks are limited to linear, quadratic, exponential equations with integer exponents, square root, cube root, piecewise, and exponential functions. The redundancy of exponential functions here should be clarified.	Tasks are limited to simple rational or radical equations.

Another example of mismatched limits between the course sequences is with:

> *A.REI.C.6 Write and solve a system of linear equations in context.*

In this case, students in the Integrated sequence are not expected to solve a system with three equations in three variables.

Algebra I	Algebra II	Integrated I
Solve systems both algebraically and graphically. Systems are limited to at most two equations in two variables.	When solving algebraically, tasks are limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to only two variables.	Solve systems both algebraically and graphically. Systems are limited to at most two equations in two variables.

The handling of the trigonometry topics in high school presents an unclear intended coherence. The TMS have removed all notions of proof, periodicity, and trigonometric modeling (TSS F.IF.4, F.TF.5, and F.TF.8) from the Traditional and Integrated sequences. In turn, the TMS have added two standards, F.TF.C.8a and F.TF.C.8c, with no direct alignments in the TSS. The TMS have also moved part of the TSS (+) standard, F.TF.3, to TMS F.TF.A.1b. In doing so they introduce the notion of "commonly recognized angle" to the sequences. Consider the full list of trigonometry standards included in both Algebra II and Integrated Mathematics III:

<table>
<tr><th>TMS Algebra II and Integrated Mathematics III</th></tr>
<tr><td>

Cluster: Extend the domain of the trigonometric functions using the unit circle

F.TF.A.1. Understand and use radian measure of an angle.

a. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

b. Use the unit circle to find *sin* θ, *cos* θ, and *tan* θ when θ is a commonly recognized angle between 0 and 2π.

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Cluster: Prove and apply trigonometric identities.

F.TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.C.8. Use trigonometric identities to find values of trig functions.

a. Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of *sin* θ, *cos* θ, and *tan* θ to evaluate the trigonometric functions.

b. Given the quadrant of the angle, use the identity $\sin^2 \theta + \cos^2 \theta = 1$ to find *sin* θ given *cos* θ, or vice versa.

c. Given the quadrant of the angle, use the identity $\tan \theta = \sin \theta / \cos \theta$ to find *sin* θ, *cos* θ, or *tan* θ given *sin* θ or *cos* θ for commonly recognized angles between 0 and 2π on the unit circle.

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</table>

There are a few points of clarification that would be helpful to readers in understanding the intended coherence of these standards:

- The final TMS should explain the thinking behind teaching the unit circle while excluding the idea of periodicity.
- The TMS should clearly articulate any intended differences between F.TF.A.1b, F.TF.C.8a, and F.TF.C.8c.
- There are two "givens" in F.TF.C.8c ("given the quadrant of the angle" and "given sin θ and cos θ"). The standard would be clearer with one set of givens. Further, since these are commonly recognized angles, it is not clear why one would need the tangent identity to find sine when given cosine.
- The TMS should describe how any of the F.TF.C.8a, F.TF.C.8b, and F.TF.C.8c standards meet the "prove" part of the cluster intention to "Prove and apply trigonometric identities."
- The TMS could explain why F.TF.A.1b is a part of F.TF.A.1 and not F.TF.A.2.

Focus

High-quality standards establish priorities about the concepts and skills that should be acquired by students. A sharpened focus helps ensure that the knowledge and skills students are expected to learn are important and manageable in any given grade or course.

There are a few noteworthy differences in focus between the TSS and the draft TMS. In the table below, we list each of the aligned standards and provide a brief description of the shift in focus.

Grades K–3

TSS Standard	Draft TMS Standard	Comments
K.CC.1. Count to 100 by ones and by tens.	K.CC.A.1. Count to 100 by ones, fives, and tens. Count backward from 10.	TMS added the expectation to count backward from 10.
K.OA.5. Fluently add and subtract within 5.	K.OA.A.5. Fluently add and subtract within 10 using mental strategies.	TMS expanded the expectation of fluency to 10 instead of 5.
	K.MD.B.3. Identify the penny, nickel, dime, and quarter and recognize the value of each.	TMS added this standard related to money. There is no match in the TSS.
1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).		TMS have no similar standard.

1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	1.OA.C.5. Add and subtract within 20 using strategies such as counting on, counting back, making 10, using fact families and related known facts, and composing/decomposing numbers with an emphasis on making ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9 or adding 6 + 7 by creating the known equivalent 6 + 4 + 3 = 10 + 3 = 13). 1.OA.C.6. Fluently add and subtract within 20 using mental strategies. By the end of Grade 1, know from memory all sums up to 10.	TMS expect students to fluently add and subtract within 20 instead of 10.
1.NBT 1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral	1.NBT.A.1. Count to 120, starting at any number. Read and write numerals to 120 and represent a number of objects with a written numeral. Count backward from 20.	TMS added counting backward from 20.
	1.MD.B.4. Count the value of a set of like coins less than one dollar using the ¢ symbol only.	Money begins in grade 2 in the TSS.
2.OA.2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.	2.OA.B.2. Fluently add and subtract within 30 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers and related subtraction facts.	TMS expect fluency within 30, while the TSS expect 20.
	3.G.A.3. Define and recognize attributes of polygons.	There is no match in the TSS. This standard, though, lacks specificity.

Grades 4–6

TSS Standard	TMS Standard	Comments
4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	4.NF.B.4. Apply and extend previous understandings of multiplication as repeated addition to multiply a whole number by a fraction.	TMS limit to repeated addition. (This added limitation seems incongruous to the sub-standards for 4.NF.B.4.)

6.RP.3d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	6.RP.A.3d. Use ratio reasoning to convert customary and metric measurement units (within the same system); manipulate and transform units appropriately when multiplying or dividing quantities.	TMS add the limitation of working in the same system.
6.SP.5.c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	6.SP.B.5c. Give quantitative measures of center (median and/or mean) and variability (range) as well as describing any overall pattern with reference to the context in which the data were gathered.	TMS removed both interquartile range and mean absolute deviation, which are more useful measures of spread than range, in that they provide a sense of how spread out the data is. This provides a foundation for students to later make more precise interpretations of data distributions, with, and without, clustering around the mean, (e.g. normal distributions).

Grades 7–8

The TMS have removed all mention of two-way tables from grades 7–8. Two TSS that deal with two-way tables, S.ID.5 and S.CP.4, have been postponed to the fourth-year Statistics course.

TSS Standard	TMS Standard	Comments
7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	7.SP.B.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers. *For example, the mean height of players on the basketball team is 10 centimeters greater than the mean height of players on the soccer team; on a dot plot or box plot, the separation between the two distributions of heights is noticeable.*	TMS removed variability from this standard.
7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. 7.SP.8a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	8.SP.B.4. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for	These three TSS have been collapsed into one and moved to grade 8 in the TMS.

7.SP.8b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.	which the compound event occurs. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.	
7.SP.8c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*		This TSS has no match in the TMS.
8.SP.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?		Unmatched in the TMS. Two-way tables have been deemphasized in the TMS for grades 8–11, postponed until the fourth-year Statistics course.

Grades 9–11

The high schools standards offer many different perspectives we might consider when we think about focus. In the tables and narratives that follow we will:

- Indicate the standards intended for all students that are unique to either set.
- Indicate which TSS standards intended for all students exist beyond the first three years in the TMS.
- Indicate which TSS (+) standards are found in courses beyond grades 9–11.
- Indicate which TSS (+) standards are nowhere in the TMS.
- Indicate shifts in focus that occur within standards.
- Compare and highlight the differences between the scopes of the two sequences.

In some cases there are standards between the two sets that have no match. Consider the following TSS non-(+) standards that are missing from the TMS:

TSS	Comments
N.RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	This standard addresses closure for the set of rational numbers and opens the door to a discussion of closure for the set of real numbers. It is very conceptual. Deemphasis in this area makes it less likely that students will ponder these relationships, aside from the usual commutative, associative, and distributive properties. The concept of closure is fairly intuitive and will be referenced in an advanced course.
A.REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	This is a proof standard that relates to solving a system of two equations. The TMS include all of the standards that require students to solve systems so they should, intuitively, have this concept. However this standard is designed to help students understand how the procedure for solving a system works and would be particularly helpful when they work to solve systems of three equations in Algebra II.
G.GPE.2 Derive the equation of a parabola given a focus and directrix.	The only conic section the whole TMS (including Precalculus) recognize is the circle, while the TSS require equations for both circles and parabolas for all students. The TMS expect students to find the equation of a circle given a radius and the center (G.GPE.1) but do not address the parabola. The TSS add equations for ellipses and hyperbolas in the (+) standards, which is also not addressed anywhere in the TMS.
G.GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	This standard expects a visual understanding of cross-sections of 3-D figures and provides an opportunity for a hands-on experience that will allow all students to grasp this concept. This approach provides a visual connection to a later study of conic sections as well as a foundation for finding volume in integral calculus. (Note: This concept is deleted from grade 7 as well.)

These two TMS for Grades 9–11 have no TSS counterpart:

Draft TMS	Comments
F.TF.C.8a. Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of sin θ, cos θ, and tan θ to evaluate the trigonometric functions.	There is no match in the TSS. See the Coherence section for further discussion.
F.TF.C.8c. Given the quadrant of the angle, use the identity tan θ = sin θ/cos θ to find sin θ, cos θ, or tan θ given sin θ or cos θ for commonly recognized angles	There is no match in the TSS. Note: TMS add an emphasis on "commonly recognized angles" at this level, which is addressed in the (+) TSS: F.TF.3. See the Coherence section for further discussion.

In some cases there are high school standards from the TSS intended for all students (not the (+) standards) that are matched only in courses beyond the Algebra and Integrated course sequences.

These standards represent expectations of TSS students during the first three years of high school but are not a part of the same in the TMS. The table provides the standard by code along with comments about what Tennessee standards currently would not include:

TSS	Comments
F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \neq 1$.*	This standard recognizes that sequences are functions that can be defined recursively. The TMS have eliminated the concept of recursive generation of sequences (see F.BF.2). However, the role of sequences as functions is not completely neglected, as F.LE.2 is included in the TMS verbatim. This standard is addressed in the fourth-year TMS Bridge Math and in Precalculus.
F.TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	This standard addresses modeling with trigonometric functions. The TMS have removed the concept of periodicity from the grades 9–11 standards. This one focuses on the graphic representation. Note that for F.IF.4, periodicity was deleted, and for F.IF.7e, the part addressing trigonometric functions was also deleted.
S.ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	These non-(+) TSS are both addressed only in the TMS fourth-year Statistics course. They address relative frequencies and interpretation of two-way frequency tables. The TMS are consistent in eliminating two-way tables, which can be an important connection for students between statistics and probability. Frequency is addressed in grade 7 in the TMS but eliminated in the high school standards for grades 9–11. (Note: Two-way tables are deleted from the grade 7 and 8 statistics standards as well. For more information, see the grades 7–8 table in this section of the report.)
S.ID.6b. Informally assess the fit of a function by plotting and analyzing residuals.	This standard, requiring using residuals to informally fit a function to its data, is addressed in the fourth-year TMS Bridge Math and is extended to include regression equations in Precalculus.
S.IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.6. Evaluate reports based on data.	Both of these non-(+) TSS are addressed in the fourth-year Statistics course. The first requires the use of simulations to gather data and compare two treatments. The second expects that students can evaluate reports based on data. Both emphasize the importance of scrutiny in reading and interpreting data.

	It is important to note that S.IC.1 and S.ID.9 are both addressed in the TMS for grades 9–11, which support the concepts missing with the elimination of these standards.

This indicates, as we mentioned earlier in this report, that it will take at least five high school courses for a student to see all of the TSS non-(+) standards that the TMS address.

There are 55 (+) standards in the TSS. Two of them overlap somewhat with the grades 9–11 courses, G.SRT.11, requiring students to apply the Laws of Sines and Cosines, and F.TF.3, focusing on commonly recognized angles. Both are related to trigonometry and both have a partial alignment to the TSS:

TSS	Draft TMS	Comments
F.TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number.	**AII(MIII).**F.TF.A.1b. Use the unit circle to find *sin* θ, *cos* θ, and *tan* θ when θ is a commonly recognized angle between 0 and 2π.	These TMS do not address the TSS (+) aspects of π–x, π+x, and 2π–x in terms of their values for x, where x is any real number. The TMS bring in part, but not all, of this TSS (+) standard. *This is the only match for this TSS requiring knowledge of special right triangles and their angles (30°, 45°, 60°, 90°). The TMS, in contrast, relate finding sine, cosine, and tangent of these "common" angles to the unit circle. (Note: The unit circle is not related to periodicity and trigonometric functions in the TMS.)*
G.SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	**G(MII).**G.SRT.C.8b. Recognize and use the Law of Sines and the Law of Cosines to solve triangles in applied problems.	This TSS (+) standard also appears again in TMS Precalculus where it is then addressed with exactly the same wording as in the TSS. In the TMS, understanding the laws comes after using them and only outside of the sequenced courses.

The following 47 TSS (+) standards can be found in fourth-year courses as indicated below:

TMS Course	TSS (+) Standards	Included Topics
Precalculus	N.CN.3, N.CN.4, N.CN.5, N.CN.6, N.CN.8, N.CN.9, N.VM.1-12, A.APR.7, F.IF.7d, F.BF.1c, F.BF.4bcd, F.BF.5, F.TF.4,6-7, F.TF.9, G.SRT.9-10, G.SRT.11	Complex numbers, vectors, matrices, the Binomial Theorem, Fundamental Theorem of Algebra, function composition, inverse functions, trigonometric functions, and prove Laws of Sines and Cosines.

Statistics	S.CP.8-9, S.MD.1-5, S.MD.7	Multiplication rule for compound events, permutations and combinations, and expected value.
Applied Math Concepts	S.MD.2, S.MD.5-7, S.CP.9	Expected value, probability distributions, outcomes and decisions, and permutations and combinations.
Bridge Math	F.IF.3	Sequences as functions.

And finally, these six TSS (+) standards are not addressed in any course in the TMS:

TSS (+) Standards not in the Draft TMS
A.APR.5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
A.REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
A.REI.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 X 3 or greater).
G.C.4. (+) Construct a tangent line from a point outside a given circle to the circle.
G.GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
G.GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

A change of wording in a standard, before adoption, can shift the focus or emphasis of the original TSS. In some cases the shift is minimal and in others, more consequential. The following table indicates examples of shifts in focus that occur within standards and provides some commentary on the differences:

TSS	Draft TMS	Difference
N.CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**AII(MII).**N.CN.A.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers, and to divide complex numbers by numbers of the form *a* + *bi* where *a* = 0 and *b* is a non-zero real number.	The TMS add division of complex numbers.
A.SSE.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	**AI(MII).**A.SSE.B.3b. Complete the square in a quadratic expression in the form $Ax^2 + Bx + C$ where A = 1 to reveal the maximum or minimum value of the function it defines.	TMS limit the leading coefficient to be equal to one. *This limitation reduces the rigor of the original TSS.*
A.SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve	**AII(MIII).**A.SSE.B.4. Recognize a finite geometric series (when the common ratio is not 1), and use the	TMS remove deriving the formula. *This change deemphasizes the conceptual understanding of the sum*

problems. *For example, calculate mortgage payments.*	sum formula to solve problems in context.	*formula that students are required to use.*
F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	**AI(AII)(MI)(MII)(MIII).**F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	Periodicity is missing in the TMS. This modeling standard is never applied to trigonometric functions in the TMS. *None of the uses of this standard in the TMS address the concept of periodicity. (Note: This TSS also has partial alignments in Precalculus course standards.)*
F.IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**AII.**F.IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior. **MIII.**F.IF.C.7d. Graph exponential and logarithmic functions, showing intercepts and end behavior.	Trigonometric functions are missing in this TMS. *In addition it is not clear why the MIII version has a different coding for the standard.*
F.BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	**AII(MI).**F.BF.A.2. Write arithmetic and geometric sequences with an explicit formula and use them to model situations.	The recursive expectation is removed from the TMS.
G.C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	**G(MIII).**G.C.B.5. Find the area of a sector of a circle in a real world context.	The TMS remove arc length and derivation of the area of a sector. Arc length is not included in the 9–11 TMS. *This TMS is in the cluster titled, "Find arc lengths and areas of sectors of circles." Arc length would be assumed but it is not specifically addressed in the TMS until Precalculus.*
S.ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	**AI(MI).**S.ID.A.1 Represent single or multiple data sets with dot plots, histograms, stem plots, and box plots.	The TMS added stem plots. *This change has little consequence in the TMS.*

The differences between the grades 9–11 sequences are somewhat discrete and usually minor, with a few exceptions. In many cases the difference between the sequences is that a concept is split over a different time span, two versus three years or one versus two. Most of these issues are explained in the Scope and Clarifications. Nearly all of the more significant gaps between the two sequences deal with algebra and functions. Some differences (as shown below) are significant and should be revisited.

- **A.SSE.B.3c:** This standard is addressed in Algebra I and II but only shows up in the Integrated sequence in Math I. This would not allow for rational exponents in the Integrated sequence. These two sequences will have different overall expectations of scope for this same standard, with the lower expectation in the Integrated sequence.
- **A.CED.A.2 and A.CED.A.4:** The difference in the treatment of these in the two sequences is considerable, in that this standard is specifically addressed only in the first year of the Traditional sequence and in all three years of the Integrated. Given the types of functions covered in the courses in the different years, the Traditional students will likely experience a lower expectation for these standards.
- **A.REI.B.4b:** Even though Math II addresses knowledge of, and operations with, the *a + bi* form for complex numbers (N.CN.A.1 and N.CN.A.2), the TMS requirement for the Integrated sequence has deleted the last part of the standard requiring students to write complex solutions to quadratic equations in *a + bi* form. The Algebra II version of the standard maintained the requirement that solutions be written in *a + bi* form. The Integrated sequence has a lower expectation.
- **A.REI.C.6:** While Algebra I and Math I are limited to two equations with two unknowns, the scope in Algebra II includes three equations with three unknowns. There is no requirement in the TMS Integrated sequence to match the Algebra II requirement. With no expectation to address this standard in other Integrated courses, the scope indicates a lower expectation for that sequence.
- **F.IF.C.7a:** This TMS is addressed only in Algebra I in the Traditional sequence but is addressed with identical scope in all three of the Integrated courses. It is made clear in the Scope and Clarifications that Math I will address linear and Math II will address quadratic graphs. It is not clear how this standard would be addressed differently in Math III, where there are no limits.
- **F.IF.C.7b:** This TMS requires knowledge of functions that go beyond the scope of Algebra I (square root, cube root, piecewise, step, and absolute value). The Scope and Clarifications state that there are no limits in Algebra I. However, for Algebra I students these functions would be considered "more complicated cases" and would be graphed using technology. This means that the Traditional sequence may not match the Integrated for this standard. The application of this standard in Math II and III would exceed the rigor in Algebra I, since those students in Math III, for example, would have had experiences with these functions and would be required to graph without technology.
- **S.ID.B.6:** The TMS address this standard in Algebra I and II and in all three of the Integrated sequence courses. The Scope and Clarifications for Math III specifically require polynomial and logarithmic functions for this standard, which are not required in the Scope and Clarifications for the Traditional sequence. The Traditional courses have a lower expectation here.

Specificity

Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive. Those that maintain a relatively consistent level of precision are easier to understand and use. Those that are overly broad or vague leave too much open to interpretation, while atomistic standards encourage a checklist approach to teaching and learning.

On the whole, the draft TMS are specific according to the Achieve criteria. In some cases the TMS combine TSS standards into a single standard; in other cases they split one TSS into separate TMS standards. Neither of these actions greatly impacts the overall specificity of the standards.

For an example of multiple TSS being collapsed into one, consider TMS 5.NBT.A.3:

TSS	Draft TMS
5.NBT.3. Read, write, and compare decimals to thousandths. 5.NBT.3b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	5.NBT.A.3. Read and write decimals to thousandths using standard form, word form, and expanded form (e.g., the expanded form of 347.392 is written as 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000)). Compare two decimals to thousandths based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship.

There are also instances when the TMS split a TSS standard into multiple standards, such as in 7.EE.B.3:

TSS	Draft TMS
7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.	7.EE.B.3. Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers presented in any form (whole numbers, fractions, and decimals). 7.EE.B.3a. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. 7.EE.B.3b. Assess the reasonableness of answers using mental computation and estimation strategies.

There are numerous instances where the TMS turn a TSS example into an expected part of the standard itself. This creates a further level of specificity, as what the TSS intend as an illustration becomes a required expectation. This may become too prescriptive, and we recommend following up on the difference. Consider the following example that appears to change the TSS intention to be more prescriptive:

TSS Grade 4	Draft TMS Grade 4
TSS 4.NF.3c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	4.NF.B.3c. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.

There are also instances of repeated standards in the Integrated sequence. For example, F.IF.C.7b and F.IF.C.7c are repeated in Integrated II and Integrated III with exactly the same scope.

Clarity/Accessibility

High-quality standards are clearly written and presented in an error-free, legible, easy-to-use format that is accessible to the general public. By this definition, **the TMS are generally clear and accessible.** There are, however, a few potential issues to consider. It is evident that the writers of the draft TMS have carefully considered the wording of the standards and have subsequently rephrased many TSS standards in an effort to add clarity. Consider the example below:

TSS Grade 1	Draft TMS Grade 1
TSS 1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	1.G.A.1. Distinguish between attributes that define a shape (e.g., number of sides and vertices) versus attributes that do not define the shape (e.g., color, orientation, overall size); build and draw two-dimensional shapes to possess defining attributes.

At some points in the TMS, rewording makes the standards less precise, such as in the case of 2.G.1 where a limitation is added in the TMS version that clouds the clarity of the expectation. Also consider 6.EE.5, where “Understand solving an equation or inequality as a process of answering a question…” is replaced with “Understand solving an equation or inequality using substitution….” The new rephrasing subtly shifts the focus from understanding what it generally means to solve an equation to focusing only on using substitution as a way of understanding. Both examples can be seen below:

TSS Standard	Draft TMS	Issue
2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	2.G.A.1. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Draw two-dimensional shapes having specified attributes, such as a given number of angles or a given number of sides of equal length (as determined directly or visually, not by measuring).	The limitation offered in this TMS is not clear. How would one "measure" the given number of angles or sides? If this limitation is related to the requirement to "draw" the shapes, which comes earlier in the sentence, that should be made clear.

6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	6.EE.B.5. Understand solving an equation or inequality using substitution to determine whether a given number in a specified set makes an equation or inequality true.	By removing the red section, the understanding of solving becomes connected to a focus on substitution.

On at least one occasion the rewording creates a progression issue:

TSS Standard	Draft TMS	Issue
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	6.G.A.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side that joins two vertices. Apply these techniques in the context of solving real-world and mathematical problems.	By removing the requirement of same coordinates (as in the TSS), this problem allows for non-horizontal or vertical segments. Solving these kinds of problems requires methods that are beyond grade 6.

In an effort to improve clarity, the TMS have added or modified numerous examples throughout. This can be helpful, but should also be done with care, as examples can become a primary way to interpret a standard. There are numerous issues of clarity with the new and modified examples. Here are some to consider:

Draft TMS Standard	Issue
6.EE.A.4. Identify when expressions are equivalent (i.e., when the expressions name the same number regardless of which value is substituted into them). *For example, the expression 5b + 3b = (5 +3) b = 8b.*	The TMS example is not an expression.
6.EE.C.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another. *For example, Susan has $1 in her savings account. She is going to save $4 each week. How much will she save weekly?*	The question "She is going to save $4 each week. How much will she save weekly?" is trivial and does not address the standard.
7.EE.A.2. Understand that rewriting an expression in different forms in a problem context can provide multiple ways of interpreting the problem and how the quantities in it are related. *For example, students understand that a 20 percent discount is the same as finding 80 percent of the cost (.80c).*	The standard is about different forms of an expression. The example is not.
3.NF.A.2b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number on the number line. *For example, 5/3 is the quantity you get when*	The new example does not address locating the fraction on the number line.

combining 5 parts together when the whole is divided into 3 equal parts.	
4.OA.A.2. Multiply or divide to solve contextual problems involving multiplicative comparison and distinguish multiplicative comparison from additive comparison. *For example, school A has 300 students and school B has 600 students: school B has two times as many students is multiplicative comparison; school B has 300 more students is additive comparison. (See Table for Addition and Subtraction Problem Types and Multiplication and Division Problem Types.)*	This example might be clearer as "For example, school A has 300 students and school B has 600 students: to say that school B has two times as many students is an example of a multiplicative comparison; to say that school B has 300 more students is an example of additive comparison."
5.NF.B.4a. Interpret the product a/b x q as a x (q ÷ b) (partition the quantity q into b equal parts and then multiply by a). Interpret the product a/b x q as (a x q) ÷ b (multiply a times the quantity q and then partition the product into b equal parts). *For example, use a visual fraction model or write a story context to show that 3/4 x 16 can be interpreted as 3 x (16 ÷ 4) or (3 x 16) ÷ 4. Do the same with 2/3 x 4/5 = 8/15. (In general, a/b x c/d = ac/bd.)*	The example in this question could be clearer. Consider the phrasing in the Progressions for the TSS for Mathematics: Number and Operations – Fractions, 3-5 (p.19)[1] $\frac{1}{3} \times 5$ is one part when 5 is partitioned into 3 parts, so $\frac{4}{3} \times 5$ is 4 parts when 5 is partitioned into 3 parts.
5.NF.B.5a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, the product of 1/2 and 1/4 will be smaller than each of the factors.* 5.NF.B.5b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relate the principle of fraction equivalence a/b = (n x a)/(n x b) to the effect of multiplying a/b by 1.	TSS 5.NF.5 is split into three separate TMS standards. This new example for 5.NF.B.5a blurs the distinction between it and 5.NF.B.5b.

Similarly, some of the elements in the high school Scope and Clarifications lack precision:

TMS Standard	TMS Scope and Clarifications	Issue
Ai(AII)(MII).F.IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions.	For example, identify rate of change in functions such as $y = 2^x$, $y = (1/2)^x$, $y = 2^{-x}$, $y = (1/2)^{-x}$. (Algebra II, p.113).	The connection here to rate of change is unclear. Is the intention percent rate of change?

[1] Available at http://math.arizona.edu/~ime/progressions/.

AI(AII)(MI).F.LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.	For example, the equation $y = 5000 (1.06)^x$ models the rising population of a city with 5000 residents when the annual growth rate is 6 percent. What will be the effect on the equation if the city's population is 7,000 instead of 8,000? (Algebra II, p. 115)	Is the intention here to ask about changes to the initial population?
AI(AII)(MII).A.REI.B.4. Solve quadratic equations and inequalities in one variable. a. Use the method of completing the square to rewrite any quadratic equation in *x* into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions.	Note: Solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials (cluster A-APR.B). Cluster A-APR.B is formally assessed in Algebra II. (Algebra I, p.100)	This is unclear. Should students be expected to solve quadratics by factoring in Algebra I? Also, note that this standard includes "inequalities," but only does so in the Algebra I version of the standard. This should be revisited.

Other issues of clarity that should be addressed include:

- There are standards in the TMS that are used multiple times across the course sequences that unnecessarily include slightly different wordings. For example, consider the version of A.CED.A.1 addressed in Algebra I, Math I, and Math II compared to the version of the same standard in Algebra II and Math III. All courses could use the same standard, with the differences pointed out in the Scope and Clarifications. This example, and others, should be revisited.
- Although this review does not include a review of content in the fourth-year courses, we noticed that there are often very different standards with the same standard identifier. For example, there are at least three versions of S.ID.A.1. One version is shared between the Traditional and Integrated course sequences. The other two versions are different from that, and different from each other, and are found in Bridge Math and Statistics.
- The precision and modeling practices are listed as MP.6 and MP.4 in one part of the document and reversed in another. This is also the case with MP.5 and MP.7, which are exchanged in the same way. The coding should consistently be applied across the TMS document.

- In the high school Geometry standards there are occasions where the TSS requirement to "prove" a theorem is changed to "recognize" or "identify." An example of this was mentioned earlier in G.SRT.11, where "understand and apply" was changed to "recognize and use." In this case it is not clear what it actually *means* to "recognize" the Laws of Sines and Cosines. The meaning should be made clear or the wording changed.

These issues could be readily addressed in the next draft of the TMS.

Measurability

Standards should focus on results rather than the processes of teaching and learning. They should make use of performance verbs that call for students to demonstrate knowledge and skills, with each standard being measurable, observable, or verifiable in some way.

The K–8 TMS reflect a comparable level of measurability to that of the TSS. The high school standards, having been aligned to two course sequences for the first three years, helpfully provided the Scope and Clarifications for those standards that cut across courses. With a few corrections, those additional supports will help clarify the measurability of the standards.

Summary

The math standards work group has clearly done a great deal of work to thoughtfully produce a highly rigorous set of standards. In grades K–8, the alignment to college- and career-ready expectations and research is very strong. The one key exception for focus is in the handling of Statistics in grades 6–8. Regarding clarity, there are specific standards that would benefit from wording changes. We hope that this detailed report and the information in the accompanying chart will help to this end.

For high school, there is a strong overlap with the overall set of high school college- and career-ready standards, including alignment with the expectations set by Tennessee's current standards. With respect to standards expected for all students, the TMS for grades 9–11 lack a few of the conceptual standards. There are a few topics that are simply not found in the TMS, such as the cross-sections of three-dimensional figures and deriving the equation of a parabola using the focus and directrix. Additionally, Tennessee students will have to take the fourth-year Statistics course to learn some of the content expected of all students under Tennessee's current standards. Similarly, students will have to take the fourth-year Precalculus course to work with periodicity and model with trigonometry, also expected of all students under Tennessee's current standards. By selecting just one fourth-year course, Tennessee students will miss out on a few of the standards that are considered important to prepare all students for postsecondary education and careers.

The specific wording of some of the high school standards in the TMS (such as from "prove" to "use") sets a lower expectation than benchmark college- and career-ready standards. Additionally, while this review identified other issues of concern with respect to coherence, focus, specificity, and clarity, they can, for the most part, be readily addressed.

There are a few notable differences between the expectations of the Traditional and Integrated sequences in the high school TMS for grades 9–11. Details are provided in the Focus section of this report and should also be addressed.

This review of the high school TMS indicates some issues of consistency with both the coding of standards across all courses and the expectations between the two grade 9–11 sequences within the TMS. Resolving these issues will provide clear guidance to Tennessee educators to inform their decisions about instructional materials and the best ways to prepare students. They will also be able to share and gain insights with educators using comparably rigorous standards in other states across the country.

Key Recommendations for Tennessee's Draft Mathematics Standards

The draft TMS are generally rigorous, coherent, and focused. This finding is especially true for grades K–8, with only a few exceptions, which are detailed in this report and in the accompanying alignment chart. To ensure the final TMS reflect the highest levels of rigor for educational standards, the Standards Review Committee may consider the following recommendations.

1. **This analysis has uncovered a few noteworthy gaps in content alignment between the TMS and the TSS. Consider the implications of these gaps and whether they require further scrutiny.**

Gaps in the alignment between the TSS and the current draft TMS may result in breaks in coherent progressions or a shortage of focus or rigor in the TMS. The accompanying side-by-side chart identifies all gaps in the alignment between the TMS and the TSS by highlighting in yellow any cell containing a standard that has no full or partial match. To identify TMS standards that partially align with the TSS, the cell is highlighted in grey. Red font is used to draw attention to differences in the two standards and/or words or phrases that are referenced in the commentary. In the K–8 alignments, there are very few yellow-highlighted cells. In K–8 we found only three TMS that have no TSS counterpart (all three are found in grades K–3). These add specific TMS expectations related to the use of money in kindergarten and grade 1 and to the attributes of polygons in grade 3. There are about 20 TMS in grades K–5 with partial alignment and an additional seven in grades 6–8. Those can be found with an explanation for each in the accompanying chart.

For the TSS in grades K–8, there are six standards that have no TMS counterparts. Those standards include one in grade 1 and the remaining five in Geometry and Statistics and Probability for grades 7–8. In addition to these there are several TMS K–8 standards that have a partial alignment to the TSS. In some cases the difference is negligible but in others there is a significant difference that affects rigor, coherence, or focus. For example, consider this grade 1 alignment:

TSS	TMS
1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	1.NBT.C.4. Add a two-digit number to a one-digit number and a two-digit number to a multiple of ten (within 100) using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

The TMS counterpart is a match with the first part of the TSS but eliminates the conceptual requirements "relate … and explain" and "understand." This amounts to a sizeable reduction in rigor when compared to the TSS.

For high school standards, we have to consider both the standards for all students as well as the subset of the TSS that are designated for students who plan to study or work in math-related fields (the (+) standards). When we compare the TSS high school standards intended for all students (the non-(+) standards) with the TMS for grades 9–11 (Algebra I-Algebra II-Geometry, or Math I-Math II-Math III), we find that the topics of 11 TSS high school standards are not found in the skills and understandings of the Traditional or the Integrated sequences.

When we consider the full set of TSS high school standards, including the (+) standards, we find that the **fourth-year courses include nearly all of the TSS topics. However, there is no single fourth-year course that addresses all of the missing TSS topics intended for all students in the TMS for grades 9–11.**

2. **There are frequent instances where the TMS have adopted, but slightly modified, a standard from the TSS and in doing so have lowered the level of rigor, coherence, or clarity of the standard, particularly in high school. Consideration should be given to the consequences of these changes in wording.**

There are several instances in the high school TMS where there are slight changes in wording as compared to the TSS. In many cases those changes do not significantly change the focus or rigor of the TSS counterpart. For example consider this TMS, which is used in five of the six sequenced courses and also addressed (verbatim) in the Bridge Math course:

TSS	Draft TMS Sequence Courses	TMS Bridge Math
N.Q.2. Define appropriate quantities for the purpose of descriptive modeling.	**AI (AII)(M1)(MII)(MII).**N.Q.A.2. Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.	N.Q.A.2. Define appropriate quantities for the purpose of descriptive modeling.

In some instances, however, a minor change in wording carries more significant ramifications. Consider the following comparison where the TMS changed the expectation from “prove … and use …” to just “use” polynomial identities:

TSS	Draft TMS Sequence Courses
A.APR.4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples*.	**AII (MIII).**A.APR.C.4. Use polynomial identities to describe numerical relationships.

Note: A different example than this one is provided in the Scope and Clarifications for Algebra II.

Unfortunately, at the high school level, changes like this one often reduce the rigor of the intentions of the TSS. This TMS eliminates the need to prove the polynomial identities students are asked to use, taking the underlying belief that they are true (i.e., a foundational understanding of the truth of those identities) out of the picture.

We provide many more comments in the accompanying chart to highlight these differences in wording and also to reference new, or modified, examples or elements of the section of the high school standards. We hope these comments can be used to identify differences that might be problematic for users of the TMS.

3. This report and the accompanying alignment chart point out a few breaks in the coherence of mathematical progressions in the TMS standards. Examine the indicated progressions and consider strengthening connections.

For example, transformations are taught in grade 8 (see TMS 8.G.2) but are not connected to similarity and congruence (see TSS 8.G.2 and 8.G.4). In addition, there are instances where the connections between cluster headings and the standards that follow are fragile.

Another type of coherence issue (using a procedure before understanding it) can be found in a high school example where the Laws of Sines and Cosines are addressed in Geometry and Math II and then in Precalculus. In the TMS, students are asked to "recognize and use" in grades 9–11 and then to "prove and use" and "understand and apply" the laws in the fourth-year course. This means that understanding the laws is not required until the Precalculus course, for those who take it, which is about two years after students have used them.

These kinds of gaps in the coherence of the standards may cause confusion and inhibit student understanding. More details about gaps in coherence can be found in the Coherence section of this report, as well as in the comment column of the accompanying chart.

4. There are some slightly different expectations between the Traditional and Integrated sequences. Consider all inconsistencies between the two high school sequences. Review the course standards and the Scope and Clarifications sections for each to make sure they are consistent.

Creating the limits and examples for two parallel pathways for grades 9–11 is noteworthy and helpful, but as we compared the two sequences with the TSS, we found some inconsistencies between them. Some examples are outlined in this report. We recommend that Tennessee continue to examine the TMS for other inconsistent treatment of the standards across the two sequences.

5. There are issues with the coding of standards in the TMS. Revise the codes used for standards where indicated in this report and the accompanying alignment chart.

In examining the TMS, we found the same coding schema is used for standards in multiple courses and levels. Sometimes they are for the same or related standards and sometimes not. Consider, for example, the use of the code, N.RN.1, for standards for grades 9–11 and also in Bridge Math:

TMS Algebra II	TMS Math II	Bridge Math
AII.N.RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	**MII.**N.RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	N.RN.A.1. Use rational and irrational numbers in calculations and in real-world context.

In this example the standards have the same code and, even though they are not exactly the same, are related.

In some cases, the same code is used for different standards in different courses with no clear relationship between those standards. For example, consider N.Q.A.3:

TMS Algebra I	TMS Math I	Bridge Math
AI.N.Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**MI.**N.Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	N.Q.A.3. Solve problems involving evaluation of exponential functions, for example applications involving simple and compound interest.

The same code is used in Bridge Math, but that standard does not align with those of the same code in the sequenced courses. This may become confusing during discussion of the standards among mathematics teams and may be particularly troublesome for high school teachers who happen to teach both sequence courses and Bridge Math.

There are also instances where the codes have been changed without an apparent reason. For example, in grade 8 Geometry, we found TMS labeled as 8.G.A.1, 8.G.A.2, 8.G.A.4, 8.G.B.6, 8.G.B.7, 8.G.B.8, and 8.G.B.9. There are no standards labeled with 8.G.A.3 or 8.G.A.5. Yet in the alignment we found that TMS 8.G.A.2 and 8.G.A.4 were exact matches for TSS 8.G.3 and 8.G.5, respectively:

TSS	Draft TMS
8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	8.G.A.2. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	8.G.A.4. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

In TMS Algebra II F.IF.C.7e is a partial match with the TSS, F.IF.7e. However, the same standard is called F.IF.C.7d in Math III. The actual TSS F.IF.7d does not show up in the TMS for grades 9–11. See the chart below:

TSS	Draft TMS Algebra II	Draft TMS Math III
F.IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**AII.**F.IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior.	**MIII.**F.IF.C.7d. Graph exponential and logarithmic functions, showing intercepts and end behavior.

Another coding issue is in the treatment of the Standards for Mathematical Practice. On pages 9–12, the numbering and codes are different from all other places in the TMS.

A consequence of this seemingly inadvertent mismatch of the codes will lead to confusion when teachers have discussions about the standards with colleagues outside of Tennessee or when Tennessee teachers search beyond Tennessee for resources based on the codes for the standards.

Appendix: The Criteria Used for the Evaluation of College- and Career-Ready Standards in English Language Arts and Mathematics

Criteria	Description
Rigor: What is the intellectual demand of the standards?	Rigor is the quintessential hallmark of exemplary standards. It is the measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in credit-bearing college courses without remediation and in entry-level, quality, high-growth jobs. For Achieve's purposes, the Common Core State Standards represent the appropriate threshold of rigor.
Coherence: Do the standards convey a unified vision of the discipline, do they establish connections among the major areas of study, and do they show a meaningful progression of content across the grades?	The way in which a state's college- and career-ready standards are categorized and broken out into supporting strands should reflect a coherent structure of the discipline and/or reveal significant relationships among the strands and how the study of one complements the study of another. If college- and career-ready standards suggest a progression, that progression should be meaningful and appropriate across the grades or grade spans.
Focus: Have choices been made about what is most important for students to learn, and is the amount of content manageable?	High-quality standards establish priorities about the concepts and skills that should be acquired by graduation from high school. Choices should be based on the knowledge and skills essential for students to succeed in postsecondary education and the world of work. For example, in mathematics, choices should exhibit an appropriate balance of conceptual understanding, procedural knowledge, and problem-solving skills, with an emphasis on application. In English language arts, standards should reflect an appropriate balance between literature and other important areas, such as informational text, oral communication, logic, and research. A sharpened focus also helps ensure that the cumulative knowledge and skills that students are expected to learn are manageable.
Specificity: Are the standards specific enough to convey the level of performance expected of students?	Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive. Standards that maintain a relatively consistent level of precision ("grain size") are easier to understand and use. Those standards that are overly broad or vague leave too much open to interpretation, increasing the likelihood that students will be held to different levels of performance, while atomistic standards encourage a checklist approach to teaching and learning that undermines students' overall understanding of the discipline. Also, standards that contain multiple expectations may be hard to translate into specific performances.
Clarity/Accessibility: Are the standards clearly written and presented in an error-free, legible, easy-to-use format that is accessible to the general public?	Clarity requires more than just plain and jargon-free prose that is also free of errors. College- and career-ready standards also must be communicated in language that can gain widespread acceptance not only from postsecondary faculty but also from employers, teachers, parents, school boards, legislators, and others who have a stake in schooling. A straightforward, functional format facilitates user access.
Measurability: Is each standard measurable, observable, or verifiable in some way?	In general, standards should focus on results rather than the processes of teaching and learning. College- and career-ready standards should make use of performance verbs that call for students to demonstrate knowledge and skills and should avoid using those verbs that refer to learning activities — such as "examine," "investigate," and "explore" — or to cognitive processes, such as "appreciate."